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A Survey of and Evaluation Methodology for Fiber Composite Material Failure Theories

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Abstract

The long-standing problem of characterizing failure for fiber composite materials will be reviewed. Emphasis will be given to the lamina level involving nominally aligned fibers in a matrix phase. However, some consideration will also be given to laminate failure using the lamina form as the basic building block along with the concept of progressive damage. The many different lamina level theories will be surveyed along with the commitment necessary to produce critical experimental data. Four particular theories will be reviewed and compared in some detail, these being the Tsai-Wu, Hashin, Puck, and Christensen forms. These four theories are reasonably representative of the great variety of different forms with widely different physical effects that can be encountered; also for comparison, the rudimentary forms of maximum normal stress and maximum normal strain criteria will be given. The controversial problem of how many different individual modes of failure are necessary to describe general failure will receive attention. A specific and detailed methodology for evaluation of all the various theories will be formulated.

Introduction

Theories of failure for anisotropic materials have been propounded for at least the past forty years. The advent of high strength, highly anisotropic fiber composite materials has accelerated the activity and accentuated the importance of the search. The lack of agreement on a single, best theory has not been for lack of activity. If one includes all forms of theoretical failure characterization, there are probably well over one hundred different theoretical forms, sometimes applicable over widely different conditions. To put some scope and limits on the present considerations, only reasonably comprehensive theories (not individual mechanism theories) will be considered, meaning theories applicable to fully three dimensional states of stress and strain. That the consideration and evaluation of such theories of behavior is a difficult proposition should be self evident. Let it just be said that even in the case of isotropic materials, the theoretical characterization of failure is not a settled issue. The corresponding problem for highly anisotropic materials could reasonably be expected to be much more difficult than that for isotropy.

A sampling of some fully three dimensional theories of failure are given in Table 1. The sources for these are given in the list of references. By no means are the theories limited to the case of just 10 adjustable parameters. There are theories with 15 or more parameters. This situation immediately raises the issue of the degree of practicality for theories with large numbers of parameters. It would appear that somewhere in the range of 7-9 parameters is pushing the upper limit. There is a corresponding problem at the other end of the scale. What is the fewest number of parameters that can capture the immensely complicated interactive physical effects that occur at the threshold of failure. Additionally, in developing theories of failure one must first make the decision of the basis type, namely stress based or strain based. By far the more common are the stress based forms, and they will receive primary emphasis here.

Most of the consideration here will be given to the lamina level form of aligned fiber composites. After examining this at some length, consideration will be given to the laminate form which is used in most applications. Nevertheless the emphasis here is upon the lamina level, since it is the basic building block for most composites applications. At the lamina level, the fiber composite material will be idealized as being transversely isotropic, and all theories to be considered will be of that type. Since these are macroscopic theories of failure, the micro-scale failure mechanisms could be due to a wide array of processes related to fiber degradation, matrix degradation, interface failure and all manner of interactive complications. When matrix controlled modes of failure are designated, this will be implicitly taken to include interface failure precipitated events.

It is important to acknowledge the existence of an ongoing fiber composite failure theory evaluation program. This commendable effort, organized by Hinton and Soden [1998] and Soden, Hinton and Kaddour [1998] was initiated some years ago and is nearing completion. It considers many more theories than just the sampling shown here in Table 1 and some theories included here are not part of their evaluation. The present evaluation examination is not coordinated with the Hinton, Soden, Kaddour study for two reasons. Their evaluation interests and procedures are entirely of a 2-D, plane stress nature, whereas the present interests are entirely of a three dimensional nature. Secondly, their aim is to evaluate 2-D lamina level theories primarily from laminate level behavior. Theirs is a rather complex undertaking because of the lamina to lamina interaction that occurs in a laminate. Even though the ultimate objective must be to predict laminate level behavior, it is here felt that the most firm ground for evaluating lamina level theories is from procedures and results obtained at the same level, that of the lamina.

Four of the theories of failure behavior shown in Table 1 are selected here for examination. These are the Tsai-Wu, Hashin, Puck and Christensen theories, all expressed in terms of stresses. These four theories are quite representative of the great variety of different forms with widely different physical effects that can be encountered. Also, for comparison, the rudimentary forms of normal stress, and normal strain criteria will be considered. All four of the fully 3-D theories are at the quadratic level of representation. After outlining these theories and showing some aspects of the varied behavior, a specific evaluation methodology will be formulated. The experimental commitment needed to effect the evaluation will receive consideration. Finally, some aspects of laminate level behavior will be considered.

Specific Theories of Failure

The four theories of failure to be considered here are those of Tsai-Wu [1971], Hashin [1980], Puck (Puck and Schürmann [1998], Kopp and Michaeli [1999]) and Christensen [1997, 1998]. Table 2 shows some characteristics of these four theories. All the theories except the Tsai-Wu form are decomposed into matrix controlled modes of failure or fiber controlled modes of failure. The Tsai-Wu form is said to be fully interactive, with all effects folded into one overall mode of failure. In the present context the number of modes of failure are defined as the number of distinct branches that are in evidence in the failure envelope in stress space. These branches may or may not intersect, but if they do, the slopes are discontinuous. From Table 1 it is seen that the number of modes of failure range from one to a number equal to the number of parameters in the theory. This perhaps as much as anything illustrates the great diversity that is encountered when comparing these theories.

An even more graphic form of the differences between the theories is as shown in Fig. 1, which are examples of the failure envelopes in a sub-space of stress. The matrix controlled modes of failure are schematically shown in σ_{22}-σ_{33} space where axis 1 is always in the fiber direction. The forms for the various theories are representations taken from related references. They are not specifically calibrated to identical properties, but rather only serve to illustrate the great variety of different behaviors which are possible. A similar comparison between fiber controlled modes of failure would likely be ever more divergent, however such forms are not readily available in the literature.

Now, the four individual theories of failure will be displayed for comparison purposes. For the most part, the terminology followed will be that of the authors.

TSAI-WU Criterion

The Tsai-Wu [1971] theory is often called the tensor polynomial theory since that is exactly what it is. The stress invariants for transversely isotropic symmetry are used in a polynomial expansion up to terms of second degree. The following form is then the failure criterion

$$\begin{aligned} F_1\sigma_{11} + F_2(\sigma_{22} + \sigma_{33}) + F_{11}\sigma_{11}^{\,2} + F_{22}(\sigma_{22}^{\,2} + \sigma_{33}^{\,2}) + 2F_{12}\sigma_{11}\,(\sigma_{22} + \sigma_{33}) \\ + 2F_{23}\sigma_{22}\sigma_{33} + 2(F_{22}-F_{23})\sigma_{23}^{\,2} + F_{66}(\sigma_{12}^{\,2} + \sigma_{31}^{\,2}) = 1 \end{aligned} \tag{1}$$

where F_1, F_2, F_{11}, F_{22}, F_{12}, F_{23} & F_{66} are the seven parameters that are to be evaluated from data. Five of the parameters are evaluated directly from

$$F_1 = \frac{1}{X_1} - \frac{1}{X_1'} \;,\; F_2 = \frac{1}{X_2} - \frac{1}{X_2'}$$

$$F_{11} = \frac{1}{X_1 X_1'} \;,\; F_{22} = \frac{1}{X_2 X_2'} \;,\; F_{66} = \frac{1}{\left(\sigma_{12}^{\,Y}\right)^2} \tag{2}$$

where X_1 and X_1' and X_2 and X_2' are the uniaxial tensile and compressive failure stress magnitudes in the axial and transverse directions respectively, $\sigma_{12}^{\,Y}$ is the axial shear failure stress. The remaining two parameters, F_{12} and F_{23}, must be evaluated from more complicated stress condition testing, or they must be estimated, the latter of which is normally done, Hahn and Kallas [1992]. This Tsai-Wu theory is by far the simplest of the four theories, yet it contains all the same three dimensional terms. To the extent allowed by the form of the invariants it has all stress components as interactive with each other through (1) and (2).

HASHIN Criteria

The Hashin [1980] criteria begins with the second degree polynomial expansion in the invariants. The failure modes are then decomposed into matrix controlled and fiber controlled forms depending upon which stress components act upon the failure planes which are taken to be parallel and perpendicular to the fiber direction respectively. Also, interaction parameter F_{12} in (1) is taken to vanish. Next each mode is further decomposed into tensile controlled and compressive controlled forms, introducing 4 additional parameters. Finally, 4 separate assumptions or conditions are imposed bringing the total parameter count to 6. Thus, the final forms are composed of 4 modes of failure along with 6 parameters.
The final forms of the criteria are as follows:
Tensile Matrix Mode, $(\sigma_{22} + \sigma_{33}) > 0$

$$\frac{1}{\left(\sigma_T^{+}\right)^2}\left(\sigma_{22} + \sigma_{33}\right)^2 + \frac{1}{\tau_T^{2}}\left(\sigma_{23}^{2} - \sigma_{22}\sigma_{33}\right) + \frac{1}{\tau_A^{2}}\left(\sigma_{12}^{2} + \sigma_{13}^{2}\right) = 1 \tag{3}$$

Compressive Matrix Mode, $(\sigma_{22} + \sigma_{33}) < 0$

$$\frac{1}{\sigma_T^{-}}\left[\left(\frac{\sigma_T^{-}}{2\tau_T}\right)^2 - 1\right]\left(\sigma_{22} + \sigma_{33}\right) + \frac{1}{4\tau_T^{2}}\left(\sigma_{22} + \sigma_{33}\right)^2 + \frac{1}{\tau_T^{2}}\left(\sigma_{23}^{2} - \sigma_{22}\sigma_{33}\right) + \frac{1}{\tau_A^{2}}\left(\sigma_{12}^{2} + \sigma_{13}^{2}\right) = 1 \tag{4}$$

Tensile Fiber Mode, $\sigma_{11} > 0$

$$\left(\frac{\sigma_{11}}{\sigma_A^{T}}\right)^2 + \frac{1}{\tau_A^{2}}\left(\sigma_{12}^{2} + \sigma_{13}^{2}\right) = 1 \tag{5}$$

Compressive Fiber Mode, $\sigma_{11} < 0$

$$\sigma_{11} = -\sigma_A^{-} \tag{6}$$

The 6 parameters have been evaluated from the fiber failure normal and shear stresses in the axial direction and the matrix controlled normal and shear stresses in the transverse direction

$$\sigma^{+} \quad ; \quad \sigma^{-} \quad ; \quad \tau$$

$$\sigma_T^+ \;;\; \sigma_T^- \;;\; \tau_T$$

As seen in (3) and (4) the tensile and compressive type matrix modes of failure are differentiated by the sign of the transverse direction mean normal stress.

PUCK Criteria

The Puck criteria have evolved over a period of many years. These works by Puck, his students and colleagues are here represented by Puck and Schurmann [1998] and Kopp and Michaeli [1999]. Whereas the Hashin criteria were somewhat motivated by the Coulomb-Mohr approach for isotropic materials, the Puck procedure goes further along this direction and follows the Coulomb-Mohr procedure quite strictly, at least insofar as matrix controlled failure modes are concerned. The Puck criteria are the most complicated forms considered here, ultimately resulting in numerical procedures.

First consider the matrix controlled modes of failure involving failure planes parallel to the fiber direction with the corresponding normal and shear stresses upon the failure plane. A failure criterion as shown in Fig. 2 is taken in terms of the axial shear stress, the transverse shear stress and the normal stress due to he transverse normal stresses. This procedure introduces 7 parameters. Next, all failure plane orientations are scanned to find the failure plane orientation with the worst combination of normal and shear stresses that produces failure in comparison with the failure criterion of Fig. 2. The end result of this numerical scanning program is the generation and display of failure surfaces in stress space. Seven different modes of failure are identified by this process.

With regard to fiber controlled modes of failure, these are written, in plane stress form, with fiber direction strain, ε_{11}, as

$$\frac{1}{\varepsilon_{1T}}\left(\varepsilon_{11} + \frac{\nu_{f12}}{E_{f_1}} m_{\sigma f}\,\sigma_{22}\right) = 1$$

for tension and (7)

for compression. The two parameters are the strains to failure ε_{1T} and ε_{1C}. Symbols ν_{f12} and E_{f1} are the fiber phase Poisson's ratio and Young's modulus. The "magnification" factors $m_{\sigma f}$ are said to be known for different fiber types.

CHRISTENSEN Criteria

The procedure followed by Christensen [1997, 1998] starts with the 7 parameter polynomial expansion. Micromechanics is used to decompose the polynomial form into 2 separate criteria, matrix and fiber controlled. Then independence of both of these criteria from failure under hydrostatic pressure is imposed. This reduces the parameter count from 7 to 5.

The matrix controlled criterion is given by

$$\alpha_1 k_1\left(\sigma_{22} + \sigma_{33}\right) + \left(1 + 2\alpha_1\right)\left[\frac{\left(\sigma_{22} - \sigma_{33}\right)^2}{4} + \sigma_{23}{}^2\right]$$

(8)

where

$$k_1 = \frac{\left|\sigma_{22}^C\right|}{2} \quad , \quad \alpha_1 = \frac{1}{2}\left(\frac{\left|\sigma_{22}^C\right|}{\sigma_{22}^T} - 1\right) \quad , \quad \beta_1 = \left(\frac{\sigma_{22}^C}{2\sigma_{12}^Y}\right)^2$$

with σ_{22}^T, σ_{22}^C and σ_{12}^Y being the transverse normal stress failure levels and the axial shear stress failure level. Condition $\sigma_{22}^T \leq |\sigma_{22}^C|$ applies here.

The fiber controlled criterion is given by

$$-\alpha_2 k_2 \sigma_{11} + \frac{1}{4}\left(1 + 2\alpha_2\right)\sigma_{11}^2 - \frac{\left(1+\alpha_2\right)^2}{2}\left(\frac{\sigma_{22} + \sigma_{33}}{2}\right)\sigma_{11} = k_2^2 \tag{9}$$

where

$$k_2 = \frac{\sigma_{11}^T}{2} \quad , \quad \alpha_2 = \frac{1}{2}\left(\frac{\sigma_{11}^T}{\left|\sigma_{11}^C\right|} - 1\right)$$

and where $\sigma_{11}^T \geq |\sigma_{11}^C|$ are the fiber direction failure stresses. This fiber controlled criterion decomposes into two separate branches as seen in particular applications.

Evaluation Methodology

Probably the most contentious argument that ensues when discussing composite material failure theories is how many different modes of failure should be expected. The sampling of different theories shown in Tables 1 and 2 ranges from 1 mode of failure through 9 modes of failure. This discussion or argument probably does not have a simple, easily established answer. Arguing the merits or demerits of various theories on a conceptual basis seems particularly open-ended and non-productive. The approach to be sought here avoids this argument by seeking the critical comparison of the theories with experimental data. The question then becomes one of what type of experimental data could and can be used for such an important purpose.

The type of experiments to be used to evaluate the theories is intimately connected with the character of the experiments used to evaluate the determining parameters in each and every theory. An informal survey of experimentalists has indicated a consensus that about five independent strength property experiments are the maximum number that is practical under most circumstances. Furthermore, the most practical explicit set of experiments are those used to determine the set of failure properties given by

$$\sigma_{11}^T, \sigma_{11}^C, \sigma_{22}^T, \sigma_{22}^C, \text{ and } \sigma_{12}^Y \tag{10}$$

which are the fiber direction tensile and compressive strengths, the transverse tensile and compressive strengths, and the axial shear strength. These are the one dimensional tests that embody current, standard practice. The method for evaluating the various theories is that they should be evaluated in the immediate, near region of the five data points used to calibrate the theories. If a given theory cannot give a successful prediction of behavior in the neighborhoods of the data points used for the calibration of the theory, then it is unlikely to be generally successful in the stress state regions far removed from the data calibration points.

In selecting a criterion to be used for the evaluation, there could be several options, but one particularly attractive one, and the one followed here is that of the effect of superimposed hydrostatic pressure. For example, if the test used to calibrate a given theory were to give a value of the transverse tensile strength as σ_{22}^T, then the evaluating experiment would involve a second testing procedure to determine by how much the transverse tensile strength, σ_{22}^T, is changed by the presence of a superimposed pressure. And, the superimposed pressure should be small in magnitude so that one is probing the near neighborhood of the data point. Testing under superimposed pressure is a well developed and well validated technique that has been extensively used with isotropic materials. The technique is particularly appropriate for composite materials since the common characteristics of $\sigma_{22}^T \neq |\sigma_{22}^C|$ and $\sigma_{11}^T \neq |\sigma_{11}^C|$ are direct manifestations of the effect of and importance of the mean normal stress effect.

The pressure effect forms corresponding to the properties (10) which are to be determined experimentally are

$$\left.\frac{d\sigma_{11}{}^T}{dp}\right|_{p=0}, \left.\frac{d\sigma_{11}{}^C}{dp}\right|_{p=0}, \left.\frac{d\sigma_{22}{}^T}{dp}\right|_{p=0}, \left.\frac{d\sigma_{22}{}^C}{dp}\right|_{p=0}, \left.\frac{d\sigma_{12}{}^Y}{dp}\right|_{p=0} \tag{11}$$

where p is the superimposed pressure. After the experimental database for these are established, then any theory calibrated by the properties data (10) would then be used to predict the values for (11). Examples of this procedure will be given shortly. It may be noted that the procedure just stated corresponds to examining the first two terms of a Taylor series. The theory calibration data, (10), corresponds to the first term and the derivatives in (11), both theory and measured, correspond to the second term. It should be recognized that this proposed approach would not constitute a comprehensive evaluation, but rather would be the first step in such a direction. An overall evaluation would likely be a graduated process. For example, the possible coupling of the fiber controlled modes of failure with the axial shear stress is of potential significance and possible controversy. Nevertheless, the present approach could be an important first step through which many or most theories could be eliminated from consideration. It also should be noted that any theory with any number of parameters could be evaluated by this method. If the number of parameters were greater than 5, the extra parameters would need to be evaluated by auxiliary information beyond that of (10), as has always been done in such cases.

It is helpful to change the notation slightly before illustrating the method with examples. Write the total stress tensor as

$$\sigma_{ij} = -p\,\delta_{ij} + \tilde{\sigma}_{ij} \tag{12}$$

where p is the applied pressure and

$$\tilde{\sigma}_{ij}$$

is the difference between the total stress and the pressure induced stress. With (12) the derivatives are related by

$$\frac{d\tilde{\sigma}_{ij}}{dp} = \frac{d\sigma_{ij}}{dp} + \delta_{ij} \tag{13}$$

Rather than using the total stress derivatives in (11) for the evaluation, use the derivatives of the difference stress given in (13). Let

$$\theta_{ij} = \left.\frac{d\tilde{\sigma}_{ij}}{dp}\right|_{p=0} \tag{14}$$

The advantage of the notation in (14) is that a value of $\theta_{ij} = 0$ is indicative of a Mises-like behavior involving independence of mean normal stress.

Three examples will now be given, the first two being quite simple, but still relevant. Two of the oldest failure criteria are those imposed upon normal stress and normal strain. Take these criteria for the fiber direction stress as

$$\sigma_{11}^{C} \le \sigma_{11} \le \sigma_{11}^{T} \tag{15}$$

The derivatives in (14) are easily shown to be, from (15),

$$\theta_{11}^{T} = 1 \tag{16}$$

$$\theta_{11}^{C} = 1 \tag{17}$$

These results do not even differentiate between different composite materials with different properties. Using the data from Parry and Wronski [1982, 1985] for graphite epoxy composites there is about $\theta_{11}^T \cong -0.8$ to -1.1 and $\theta_{11}^C \cong 0.38$ to 0.42. Based upon these rather old data sets, the sign of (16) is even incorrect, while the scale of (17) appears to be too large.
Now consider normal strain in the fiber direction, as the failure criterion

$$\varepsilon_{11}^C \leq \varepsilon_{11} \leq \varepsilon_{11}^T \qquad (18)$$

It can be shown that for (18) the derivatives (14) are given by

$$\theta_{11}^T = (1-2\nu_{12}) \qquad (19)$$

$$\theta_{11}^C = (1-2\nu_{12}) \qquad (20)$$

where ν_{12} is the axial Poisson's ratio. Compared with the data of Parry and Wronski mentioned above, relation (19) appears to be of the incorrect sign.

These two examples show that this evaluation method is highly discriminating. But before any conclusions could be drawn, it would be necessary to have modern, highly reliable data. Now a more comprehensive example will be given to show that the procedure is well posed and entirely practical. Using the failure criteria of Christensen (8) and (9), the derivatives in (14) can be determined in a straightforward manner, giving

$$\theta_{11}^T = \frac{1}{2}\left(1 + \frac{\sigma_{11}^T}{\sigma_{11}^C}\right), \quad \theta_{11}^C = \frac{1}{2}\left(1 + \frac{\sigma_{11}^C}{\sigma_{11}^T}\right),$$

$$\theta_{22}^T = 2\left(\frac{1 + \dfrac{\sigma_{22}^T}{\sigma_{22}^C}}{1 - \dfrac{\sigma_{22}^T}{\sigma_{22}^C}}\right), \quad \theta_{22}^C = -2\left(\frac{1 + \dfrac{\sigma_{22}^T}{\sigma_{22}^C}}{1 - \dfrac{\sigma_{22}^T}{\sigma_{22}^C}}\right)$$

$$\theta_{12} = \frac{\sigma_{12}^Y}{\sigma_{22}^T}\left(1 + \frac{\sigma_{22}^T}{\sigma_{22}^C}\right)$$

(21)

Typical data for graphite-epoxy composites are given by

$\sigma_{11}{}^{T} = 2500$ MPa, $\sigma_{11}{}^{C} = -1600$, $\sigma_{22}{}^{T} = 70$, $\sigma_{22}{}^{C} = -200$, $\sigma_{12}{}^{Y} = 80$
(22)

Using data set (22) in the derivatives (21) gives

$$\theta_{11}{}^{T} = -.281,\ \theta_{11}{}^{C} = .180,\ \theta_{22}{}^{T} = .963,\ \theta_{22}{}^{C} = -.963,\ \theta_{12} = .743 \qquad (23)$$

It is the specific results such as in (23) that are to be compared with modern experimental measurements permitting the formal evaluation.

These examples illustrate the well grounded and basic nature of this program for composite failure theory evaluation. If an advocate of a particular failure theory should allege that it is too difficult to evaluate the derivatives in (14) then there could be cause to doubt the utility and practicality of the theory under question. The vital need for well characterized failure data will be considered in the last section.

These four theories plus the simple normal stress and strain criteria are employed here as samples, albeit significant ones. Any theory could be subjected to this evaluation methodology. It is possible to build up a picture of overall behavior through the combination of individual modes of failure, although these four theories do not illustrate that. Such a procedure has been recommended by Hart-Smith [1993], and any overall theory of behavior so formed could also be evaluated by this method.

Laminates

Treating the failure of laminates is inherently much more complicated than that of a lamina. Nevertheless, there is a widely used approach for the case of laminate failure. This is usually called that of progressive damage, and it is a natural extension of lamina level failure theory. The simplest form is that of first ply failure, but that approach is too conservative for most purposes. The decomposition of moduli type and failure type properties into matrix controlled and fiber controlled modes is the motivation for progressive damage. With strain compatibility in adjacent lamina within a laminate, the damage spreads from lamina to lamina. Matrix failure occurs at much lower strains than fiber controlled effects. So the matrix controlled degradation starts the process usually. As loading continues, the properties must be degraded selectively or collectively. Then, when fiber controlled failures occur over a sufficiently large region, structural failure follows. One could use fracture mechanics to motivate and describe a critical scale of local failure beyond which uncontrolled fracture follows. All this properties degradation process works fairly well, it is semi-empirical but well established. The transverse cracking at the lamina level can lead to delamination between lamina. Although appealing in its step by step approach, the method can become quite complicated with many pitfalls and traps. The status of the progressive damage methodology is probably best described by Rohrauer [1999] in a PhD thesis of unusual scope and gravity. He states: "The quest to ascertain what is happening inside a failed lamina and how it affects the continued existence or catastrophic destruction of the whole laminate is a continuing one. Few subjects are as confusing and complex. The search for answers has lead to voluminous publications, few if any definitive methods useful to the designer exist as of yet."

Certainly significant steps have been taken to define the lamina level damage problem in precise terms that admit extension and generalization. These works include the essentially lamina level matrix cracking problem by Hashin [1996] and Nairn and Shu [1994].

An alternative to the lamina to laminate failure sequencing through progressive damage can be seen. This would be to treat failure entirely at the laminate level and thereby obtain a complete, self contained treatment. This apparently has not been done in the past. It has more or less been taken as obvious truth that the laminate level is too difficult to approach directly and that it must be approached incrementally through progressive damage. That view not withstanding, the full problem should be approached full on, with the proper metrics and invariants there is a reasonable case to be made for optimism.

Conclusions

The overall conclusion is that it is possible and practical to compare and evaluate fiber composite material failure theories at the lamina level. This would avoid the complications of evaluating lamina level theories from behavior at the laminate level which intermixes all the assumptions and idealizations of the lamina level theory with all the assumptions and idealizations of any particular progressive damage scenario. In reality, two separate and distinct evaluations should be conducted, that for the lamina level failure theory, and that for the progressive damage treatment of laminates. The present work has been focused upon the lamina level evaluation. The specific recommendations are as follows.

i) Collect the various failure theories that are to be considered, four of them have been discussed here. Reduce them to the form that predicts the quantitative effect of a superimposed pressure upon the five basic strength characteristics for the aligned fiber lamina form.

ii) To calibrate and normalize the theories, experimentally determine the five basic strength properties for the fiber composite systems of interest. Repeat the experimental determination of the five strength properties under the effect of a superimposed pressure. Combine results from i) and ii) to evaluate the lamina level theories. It is of great importance that the experimental investigation receive the highest possible priority. The detailed evaluation will be no better than the quality and reliability of the data. An enormous effort has been expended on the development of the many different theoretical forms over many years. A commensurate effort should be dedicated to the experimental investigation in order to complete a meaningful evaluation process.

iii) As a completely separate matter, the progressive damage methodology should be more highly developed and then the various proposed steps and procedures be evaluated.

iv) As another completely separate but related matter, failure characterization should be formulated directly at the laminate level, and ultimately compared with projections and predictions obtained from lamina level theory combined with progressive damage.

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Table 1. Theories of Failure

Originator	Type	Parameters
Boehler and Delafin [1979]	Stress	10
Puck [1999]	Stress	9
Cuntze [1999]	Stress	8
Tsai-Wu [1971]	Stress	7
Hashin [1980]	Stress	6
Feng [1991]	Strain	6
Christensen [1998]	Stress	5
Christensen [1988]	Strain	4
Gosse [1999]	Strain	4

Failure Modes

	Matrix Controlled		Fiber Controlled		Total	
	No. Modes	No. Params	No. Modes	No. Params	No. Modes	No. Params
Tsai-Wu					1	7
Hashin	2	4	2	2	4	6
Puck	7	7	2	2	9	9
Christensen	1	3	2	2	3	5

Table 2. Numbers of Failure Modes and Parameters

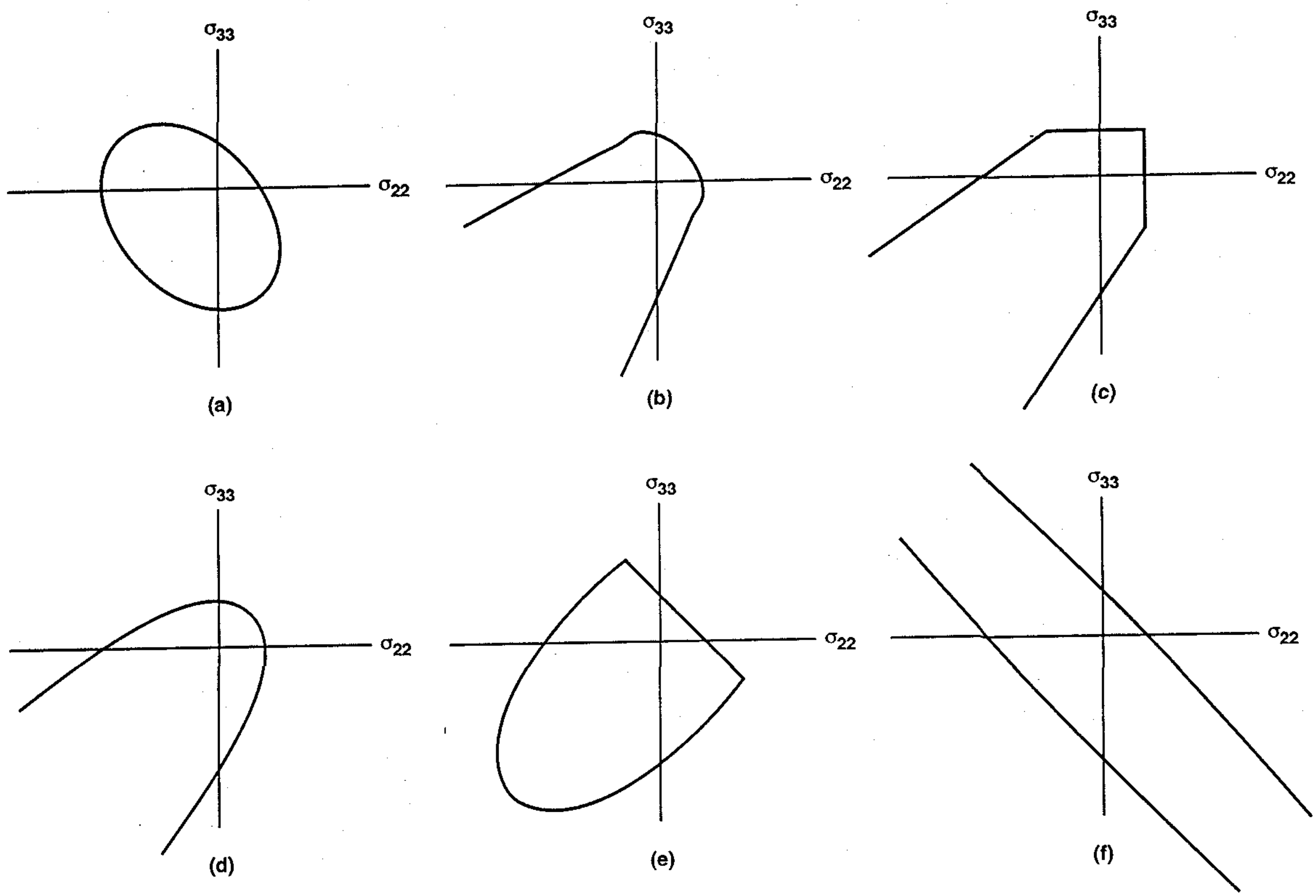

Fig. 1 Biaxial Stress Failure Examples: (a) Tsai-Wu, (b) Hashin, (c) Puck, (d) Christensen, (e) Gosse, (f) Feng

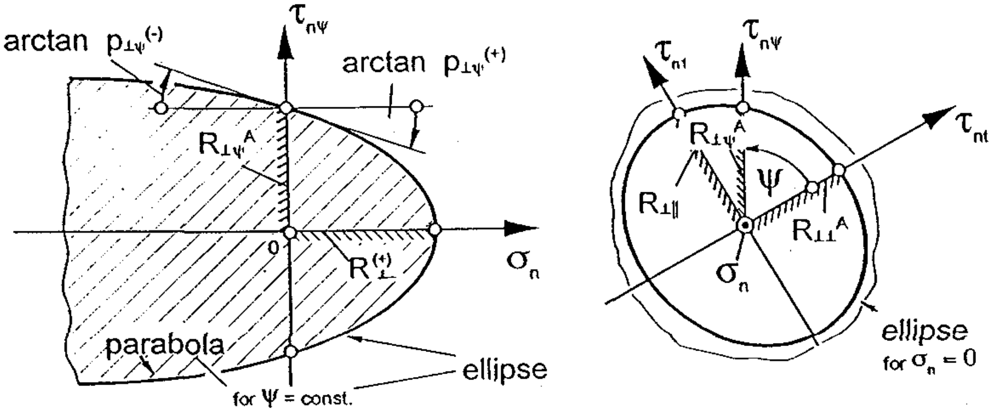

Fig. 2 Puck Theory Failure Surface